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ABSTRACT

Penalty Shoot-Outs: Before or After Extra Time?*

This paper proposes a rule to determine the winner of a soccer match which is different from the traditional penalty shoot-outs at the end of extra time. We show that games can be more attractive if penalties are shot before extra time and the outcome counts only if the tie is preserved during extra time. In general, this rule will promote offense by the team that loses the penalty shoot-outs and it will promote defense by the team that wins the penalty shoot-outs. We provide conditions on the marginal effect of offensive play in the probabilities of scoring and conceding a goal such that the proposed rule dominates the current one. Last, we determine a class of functions that satisfies these conditions. More generally, the paper shows how the ordering of tasks may affect the incentives to exert and allocate effort.

JEL Classification: Z0
Keywords: effort allocation, implicit incentives and sports economics

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1 Introduction

It is well known that an adequate provision of incentives is crucial in order to obtain maximal performance from individuals. For a long time, the Theory of Organizations has studied the merits of different explicit and implicit incentive schemes: rank-order tournaments, piece-rate compensation, team compensation, profit-sharing, promotions, career-concerns, etc. (see Prendergast (1999) for a survey). One issue that, to our knowledge, has never been analyzed is how the *ordering* of tasks affects the overall incentives of agents to perform efficiently. For example, consider a manager with two employees with heterogeneous but unknown abilities. Each employee has to work sequentially on two tasks and the manager uses promotion of the employee with highest performance at the end of both tasks as an incentive mechanism to elicit effort from them. If one task is relatively more valuable than the other and the performances of employees are publicly observed, should the manager start with the most or the least valuable one? The answer is far from evident: employees have similar incentives to exert effort in the first task, whereas their marginal value of working on the second task is high (respectively, low) when performances in the first one are similar (respectively, far apart).

In this paper, we do not attempt to provide a general theory of optimal incentive provision through task ordering. Instead, the importance of timing and ordering is acknowledged and applied to sports. There are several advantages of studying incentives in sports relative to other activities. Among others, in sports (i) rules are simple and well-defined, (ii) incentives of players are relatively uncontroversial, (iii) rule designers have clear objectives selected from a small subset of interrelated alternatives (beauty of the game, revenues generated, suspense created, etc.) and (iv) they have the flexibility to modify, at least to some extent, the rules in order to better achieve these objectives. It is therefore an ideal benchmark for a game theoretical study of incentive provision.

The paper focuses on soccer. In official elimination tournaments, teams play during 90 minutes. If, at the end of regular time, teams are tied (an event which, contrary to other sports, occurs with high probability), then they play an extra time. If the draw persists after the extra time, the winner is selected by penalty shoot-outs.[1] Extra time can be played under different rules. Traditionally, teams played extra time for a fixed length of 30 minutes. In the 1990s, most major international competitions such as the World Cup (men and women), the European Cup, the Confederations Cup, the Champions League, the Cup Winners' Cup and the UEFA Cup adopted the "golden goal" rule, where the first team to score within the 30 minutes is declared the winner.[2,3] Based on the idea that the timing of events matters for incentives, the purpose of this paper is to determine whether a rule change could help increase the excitement of extra time. More precisely, we ask the following question. Instead of shooting penalties after extra time (as it is currently the case), could it be preferable to shoot them before extra time and let them count only if the extra time is unsuccessful in breaking the tie?

Our results can be summarized as follows. First, we assume exogenous instantaneous probabilities of scoring by both teams and determine their likelihood of winning an extra time played under the golden goal rule as a function of these parameters (Proposition 1). We then endogenize the scoring probabilities. More precisely, we argue that they depend on the strategies adopted by both teams, where a more offensive strategy by a team increases both its chances of scoring and its chances of conceding a goal. Given the probabilities of victory and defeat determined previously, we then show that the common intuition holds under fairly general conditions: if penalties are shot before extra time, the team that wins (loses) the penalty shoot-outs has the greatest incentives to preserve

[1]In league tournaments, draws are permitted except in some competitions such as the Major Soccer League. However, rules are designed to decrease their occurrence. The most popular one is the "three-point victory rule" where a team that wins, ties and loses gets 3, 1 and 0 points respectively.

[2]In 2004, the European Cup has used a hybrid between the two rules, the "silver goal" rule: if a team leads after 15 minutes it is declared the winner and, if not, they play for another 15 minutes. Since then, many competitions have gone back to the fixed 30 minutes extra time play.

[3]It is widely recognized that the two main reasons for these changes are the willingness to increase the excitement of the extra time and the willingness to decrease the likelihood of reaching the penalty shoot-outs (a resolution judged "unfair" by many people).

(break) the tie and therefore plays more defensively (offensively) than if penalties are shot after extra time (Proposition 2). The final issue is then to determine whether the total level of offensive play is greater when both teams have average incentives to attack (penalties after extra time) or when one has high and one has low incentives to attack (penalties before extra time). We show that the answer to that question crucially depends on the shape of the ratio between the marginal effect of the strategy in the likelihood of scoring and in the likelihood of conceding a goal (Proposition 3). Last, we provide an example of a class of functions for the scoring probabilities such that shooting penalties before extra time is unambiguously beneficial for the attractiveness of the game.

Before presenting the model, we briefly review some literature related to our paper. Research in the sports literature about the effect of tournament design (and more specifically rewards based on rank-order vs. linear score differences) on the incentives of players or teams to exert costly effort was pioneered by Lazear and Rosen (1981). It has recently been extended by Chan, Courty and Hao (2003) in two dimensions: a dynamic framework and a preference of the public for outcome uncertainty or "suspense". Contrary to these papers, we fix the total amount of effort to be exerted and focus on its allocation between offense and defense. In this respect, we are closer to Palomino, Rigotti and Rustichini (2000) and Brocas and Carrillo (2004). The first paper studies the relative importance of effort allocation, talent, and home factor in determining the outcome of soccer matches. The second paper provides a theoretical framework to study how two rule changes, the "three-point victory" and the "golden goal", affect the allocation of effort. Empirical analyses of the effects of rule changes in the behavior of teams include Palacios-Huerta (2004) using English soccer data, Aiello and Veall (2003) using National Hockey League (NHL) and Canadian Football League data, and Abrevaya (2004) also with NHL data. None of these papers (and to our knowledge no paper in the incentives literature in general), discusses the effect of ordering on effort provision or on the choice of any other strategic variable.

2 A simple model of incentives in extra time

Two risk-neutral soccer teams of equal strength, $i \in \{A, B\}$, play an extra time. For technical simplicity, we assume that the "golden goal" rule is employed although, as we discuss below, the insights are similar under the traditional, fixed length rule. According to the golden goal rule, the first team to score one goal within the maximum time allocated for extra time wins the match. The payoffs of victory and defeat are normalized to 1 and 0 respectively. Our objective is to determine whether the order of events affects the strategies of teams. More specifically, we compare two scenarios, that we index by $k \in \{1, 2\}$. In scenario 1, teams play the extra time and, if no goal is scored, they proceed to penalty shoot-outs. Given the equal strength assumption, this means that each team's expected payoff under no goal in extra time is 1/2. This case corresponds to the system currently employed. In scenario 2, penalties are shot before extra time. However, the outcome of the penalty shoot-outs counts only if the extra time ends up with no goals. Assuming that one team (from now on and without loss of generality, team A) wins the penalty shoot-outs, the payoff of teams A and B in case of no goals during extra time are 1 and 0 respectively. Naturally, the model could be easily extended to account for a partial (rather than a full) advantage of one team in case of no goals.[4]

2.1 Scoring probabilities

We index by $t \in [0, T]$ the length of extra time and take a continuous-time approach. We denote by $\alpha\, f(t) \in (0, 1)$ and $\beta\, f(t) \in (0, 1)$ respectively the probability of scoring by team A and team B between t and $t + \Delta$ as $\Delta \to 0$. Given the continuous-time approach, the probability that both teams score at the same time is 0. We also denote $F'(t) = f(t)$ and assume that $F(T) = 1$ and $F(0) = 0$. Therefore, the total probabilities of scoring by teams A and B assuming that the rival does not score during the entire

[4]Formally, a payoff x for A and $1 - x$ for B with $x \in (1/2, 1)$. For example, this could correspond to n ($\in \{1, 2, 3, 4\}$) out of 5 penalties being shot before extra time, leaving $5 - n$ to be shot after extra time. Although theoretically interesting, this alternative seems more difficult to implement in practice.

extra time are $\int_0^T \alpha\, f(t)dt = \alpha \in (0,1)$ and $\int_0^T \beta\, f(t)dt = \beta \in (0,1)$ respectively. As developed in section 2.2 below, the probability functions α and β will crucially depend on the degree of offensive play by teams A and B, a choice variable in the model. In other words, teams will choose a strategy that will affect not only their own probability of scoring but also the probability of scoring of the rival. However, for the time being, we take them as exogenously given. Note also that, other things being equal, $f(t) = 1/T$ for all t corresponds to a constant probability of scoring over the entire period, $f'(t) < 0$ corresponds to a decreasing probability of scoring (e.g., because forwards become relatively more tired or less accurate than defenders as time elapses) and $f'(t) > 0$ corresponds to an increasing probability of scoring (e.g., because defenders become relatively more tired than forwards as time elapses). Except for symmetry ($f(t)$ is the same for teams A and B), we do not impose any structure in the functional form of $f(\cdot)$.

Given these instantaneous scoring probabilities, we can determine the likelihood that A wins the extra time, which we denote $\Pr(A)$:

$$\begin{aligned}\Pr(A) &= \int_{t_b=0}^{T}\int_{t_a=0}^{t_b} \alpha f(t_a)\,\beta f(t_b)\,dt_a\,dt_b + \int_{t_b=0}^{T}\int_{t_a=0}^{T} \alpha f(t_a)(1-\beta)f(t_b)\,dt_a\,dt_b \\ &= \alpha\,\beta\int_{t_b=0}^{T} F(t_b)\,f(t_b)dt_b + \alpha(1-\beta)\end{aligned}$$

where the first term is the probability that A scores before B does and the second term is the probability that A scores if B does not score in the entire extra time. Integrating by parts, we finally get:

$$\Pr(A) = \alpha\left(1 - \frac{\beta}{2}\right) \tag{1}$$

Using similar techniques we can determine $\Pr(B)$, the probability that B wins the extra time, and $\Pr(O)$, the probability that no team scores during extra time:

$$\begin{aligned}\Pr(B) &= \int_{t_a=0}^{T}\int_{t_b=0}^{t_a} \beta f(t_b)\,\alpha f(t_a)\,dt_b\,dt_a + \int_{t_a=0}^{T}\int_{t_b=0}^{T} \beta f(t_b)(1-\alpha)f(t_a)\,dt_b\,dt_a \\ &= \beta\left(1 - \frac{\alpha}{2}\right) \qquad (2)\end{aligned}$$

$$\begin{aligned}\Pr(O) &= \int_{t_b=0}^{T}\int_{t_a=0}^{T}(1-\alpha)\,f(t_a)\,(1-\beta)\,f(t_b)\,dt_a\,dt_b \\ &= (1-\alpha)(1-\beta) \qquad (3)\end{aligned}$$

A straightforward but nonetheless interesting observation is that, under the golden goal rule, the probability of a draw is decreasing in the likelihood of scoring by each team ($\partial \Pr(O)/\partial\alpha < 0$ and $\partial \Pr(O)/\partial\beta < 0$). Using (1), (2) and (3), we can summarize our first conclusion as follows.

Proposition 1 (Expected payoff in extra time)
Given the scoring probabilities α and β, the expected payoff V_i^k of team $i \in \{A, B\}$ under scenario $k \in \{1, 2\}$ is:

$$V_A^1 = \Pr(A) + \frac{1}{2}\Pr(O) = \frac{1+\alpha-\beta}{2} \quad \text{and} \quad V_B^1 = \Pr(B) + \frac{1}{2}\Pr(O) = \frac{1+\beta-\alpha}{2}$$
$$V_A^2 = \Pr(A) + \Pr(O) = 1 - \beta\left(1-\frac{\alpha}{2}\right) \quad \text{and} \quad V_B^2 = \Pr(B) = \beta\left(1-\frac{\alpha}{2}\right)$$

The value functions V_i^k that we derived from first principles have rather natural properties. For both scenarios, each team's payoff is increasing in its instantaneous likelihood of scoring ($\partial V_A^k/\partial\alpha > 0$ and $\partial V_B^k/\partial\beta > 0$) and decreasing in its instantaneous likelihood of conceding a goal ($\partial V_A^k/\partial\beta < 0$ and $\partial V_B^k/\partial\alpha < 0$).

At this point a remark is in order. For analytical simplicity, we have focused on extra time played under the golden goal rule, even though the traditional rule is back in force. Indeed, two complications emerge under the traditional rule, where the entire extra time is played independently of the scoring dynamics. First, a higher probability of scoring (i.e., greater values of α and β) decreases the chances of a 0-0 tie but it increases the chances of other ties (1-1, 2-2, and so on). Second, it becomes important to capture the empirical fact that teams change their strategy after scoring or conceding a goal (as in Brocas and Carrillo (2004)). At the end of the next section, we provide an intuition of why our main results should hold as well under the traditional rule.

2.2 Strategies of teams

Proposition 1 determines the likelihood of winning, tying and losing in extra time as a function of each team's probability of scoring. Naturally, these probabilities depend on the degree of offensive play of both teams, the choice variables in our model. More concretely, suppose that each team controls its own degree of offensive play. Team A chooses $a \in [\underline{c}, \overline{c}]$ and team B chooses $b \in [\underline{c}, \overline{c}]$, where higher values of a and b denote more offensive strategies. From now on we will simply call "strategy" the level of offensive play by a team. Let $\alpha(a, b)$ and $\beta(a, b)$ be the probability of scoring by teams A and B respectively, where the first argument in these functions captures the strategy of team A and the second argument captures the strategy of team B. The assumption that teams are equally strong is formalized as:

Assumption 1 $\alpha(c', c'') = \beta(c'', c')$ *for all* $(c', c'') \in [\underline{c}, \overline{c}]^2$.

It seems also natural to assume that allocating resources into a more offensive play increases both the chances of scoring and the chances of conceding a goal (subscript n denotes a partial derivative with respect to the n^{th} argument):

Assumption 2 $\alpha_1(a, b) > 0$ *and* $\alpha_2(a, b) > 0$ *(so* $\beta_1(a, b) > 0$ *and* $\beta_2(a, b) > 0$*).*

Finally, we impose conditions on the second-order and cross derivatives to have simple, unique, interior solutions:

Assumption 3 $\alpha_{11}(a, b) \leqslant 0$, $\alpha_{12}(a, b) = 0$ *and* $\alpha_{22}(a, b) \geqslant 0$ *(so* $\beta_{11}(a, b) \geqslant 0$, $\beta_{12}(a, b) = 0$ *and* $\beta_{22}(a, b) \leqslant 0$*) with at least one strict inequality.*

Playing more offensively has a positive but decreasing effect on the probability of scoring and a positive and increasing effect on the probability of conceding a goal. Concavity and convexity of the benefit and cost of the strategy are standard conditions to obtain interior solutions. Also, the marginal effect of each team's strategy is independent of the choice made by the rival. This last assumption is controversial, but there is no clear consensus among soccer fans on whether the sign is more likely to be positive or negative (see Palomino, Rigotti and Rustichini (2000) for a discussion). Imposing this assumption is not crucial for our analysis. However, it has the convenient property of ensuring uniqueness.

2.2.1 Strategies with penalties after extra time

Denote (a^*, b^*) the equilibrium pair of strategies selected by A and B when penalties are shot after extra time. First-order conditions on the expected payoff function $V_i^1(a,b)$ of team i imply:[5]

$$\left.\frac{\partial V_A^1(a,b)}{\partial a}\right|_{a^*} = 0 \quad \Leftrightarrow \quad \frac{\beta_1(a^*)}{\alpha_1(a^*)} = 1 \tag{4}$$

and

$$\left.\frac{\partial V_B^1(a,b)}{\partial b}\right|_{b^*} = 0 \quad \Leftrightarrow \quad \frac{\beta_2(b^*)}{\alpha_2(b^*)} = 1 \tag{5}$$

Given Assumption 3, second-order conditions are satisfied globally:

$$\frac{\partial^2 V_A^1(a,b)}{\partial a^2} = \alpha_{11}(a,b) - \beta_{11}(a,b) < 0$$

and

$$\frac{\partial^2 V_B^1(a,b)}{\partial b^2} = \beta_{22}(a,b) - \alpha_{22}(a,b) < 0$$

Last, each team's reaction function is independent of the choice of its rival:

$$\frac{\partial a^*}{\partial b} \propto \frac{\partial^2 V_A^1(a,b)}{\partial a \partial b} = 0$$

and

$$\frac{\partial b^*}{\partial a} \propto \frac{\partial^2 V_B^1(a,b)}{\partial a \partial b} = 0$$

where "$\propto$" stands for "proportional to."

[5] Given $\alpha_{12} = \beta_{12} = 0$, each team's marginal probability of scoring and conceding a goal is independent of the strategy of the rival. We therefore suppress the argument.

2.2.2 Strategies with penalties before extra time

More interestingly, denote (a^{**}, b^{**}) the equilibrium pair of strategies selected by teams A and B when penalties are shot before extra time, they have been won by team A, but the result counts only if the extra time ends with no goals. First-order conditions on the expected payoff function $V_i^2(a,b)$ of team i imply:

$$\left.\frac{\partial V_A^2}{\partial a}\right|_{a^{**}} = 0 \Leftrightarrow \frac{\beta_1(a^{**})}{\alpha_1(a^{**})} = \frac{\beta(a^{**}, b)}{2 - \alpha(a^{**}, b)}$$

and

$$\left.\frac{\partial V_B^2}{\partial b}\right|_{b^{**}} = 0 \Leftrightarrow \frac{\beta_2(b^{**})}{\alpha_2(b^{**})} = \frac{\beta(a, b^{**})}{2 - \alpha(a, b^{**})}$$

The local second-order condition for team A is:

$$\left.\frac{\partial^2 V_A^2}{\partial a^2}\right|_{a^{**}} = -\beta_{11}(a^{**})\left(1 - \frac{\alpha(a^{**}, b)}{2}\right) + \beta_1(a^{**})\alpha_1(a^{**}) + \alpha_{11}(a^{**})\frac{\beta(a^{**}, b)}{2} < 0$$

Using the first-order condition, we then have that a^{**} is a local maximum if and only if:

$$\left.\frac{\partial^2 V_A^2}{\partial a^2}\right|_{a^{**}} < 0 \quad \Leftrightarrow \quad 2\beta_1(a^{**}) - \beta(a^{**}, b)\left(\frac{\beta_{11}(a^{**})}{\beta_1(a^{**})} - \frac{\alpha_{11}(a^{**})}{\alpha_1(a^{**})}\right) < 0 \qquad \textbf{(C)}$$

which we call condition **(C)**. The local-second order condition for team B is:

$$\left.\frac{\partial^2 V_B^2}{\partial b^2}\right|_{b^{**}} = \beta_{22}(b^{**})\left(1 - \frac{\alpha(a, b^{**})}{2}\right) - \beta_2(b^{**})\alpha_2(b^{**}) - \alpha_{22}(b^{**})\frac{\beta(a, b^{**})}{2} < 0$$

which is always satisfied given Assumptions 2 and 3. Assuming that team A's second-order condition **(C)** is satisfied, then reaction functions are:

$$\frac{\partial a^{**}}{\partial b} \propto \frac{\partial^2 V_A^2}{\partial a \partial b} = \frac{\beta_1(a)\alpha_2(b) + \alpha_1(a)\beta_2(b)}{2} > 0$$

and

$$\frac{\partial b^{**}}{\partial a} \propto \frac{\partial^2 V_B^2}{\partial a \partial b} = -\frac{\beta_2(b)\alpha_1(a) + \alpha_2(b)\beta_1(a)}{2} < 0$$

Hence, if **(C)** is satisfied, the equilibrium strategies (a^{**}, b^{**}) are unique and given by:

$$\frac{\beta_1(a^{**})}{\alpha_1(a^{**})} = \frac{\beta(a^{**}, b^{**})}{2 - \alpha(a^{**}, b^{**})} \tag{6}$$

and

$$\frac{\beta_2(b^{**})}{\alpha_2(b^{**})} = \frac{\beta(a^{**}, b^{**})}{2 - \alpha(a^{**}, b^{**})} \tag{7}$$

From (4), (5), (6), (7), given Assumptions 1, 2 and 3, $\beta < 1$ and $2 - \alpha > 1$, we have the following conclusion.

Proposition 2 (Optimal strategies in extra time)

Assume **(C)** *holds. The strategies of teams when penalties are shot after extra time* (a^*, b^*) *and before extra time* (a^{**}, b^{**}) *are such that:*

$$a^{**} < a^* = b^* < b^{**}$$

The interpretation is simple but important. Given the equal strength assumption, if penalties are shot after extra time, then both teams have the same incentives to play offensively. If penalties are shot before extra time, then the team that has won the penalty shoot-outs has greatest incentives to play defensively in order to preserve a tie, which is enough to obtain the maximal reward. On the opposite side, the team that has lost the penalty shoot-outs has greatest incentives to play offensively, so as to break the tie and avoid getting no reward. All strategies remain at interior levels as long as there is enough concavity and convexity in the probabilities of scoring and conceding a goal. Thus, it is a priori unclear which rule maximizes the total amount of offensive play.

This result should hold also under the traditional rule, with a fixed 30 minutes length of extra time. In that case, teams will most probably change their strategy as goals are scored. However, the distortion introduced when penalties are shot before the extra time is still very similar qualitatively: by providing an initial advantage to one team, the expected proportion of extra time in which the incentives are asymmetric (one team attacks and one team defends) is increased relative to the expected proportion of extra time in which the incentives are symmetric.

3 Penalties before or penalties after extra time?

3.1 A theoretical comparison of offensive play

Suppose that the objective of the rule designer is to maximize the total amount of offensive play. In order to compare $a^* + b^*$ with $a^{**} + b^{**}$ we need to impose some more structure on the functions $\alpha(\cdot,\cdot)$ and $\beta(\cdot,\cdot)$. To keep the maximum possible level of generality, let us define:

$$g(c) \equiv \frac{\alpha_2(c)}{\alpha_1(c)} \qquad \text{and} \qquad h(c) \equiv \frac{\alpha_1(c)}{\alpha_2(c)}$$

so that $h(c) = 1/g(c)$. Assumption 3 implies that $g' > 0$ and $h' < 0$. From (4), (5), (6) and (7), we have:

$$g(a^*) = g(b^*) = 1 \qquad \text{and} \qquad g(a^{**}) \times g(b^{**}) = 1 \tag{8}$$

Let us suppose now that $g(a^{**}) = \gamma$ and therefore $g(b^{**}) = 1/\gamma$. Since $g' > 0$ and, according to Proposition 2, $a^{**} < b^{**}$, then $\gamma < 1$. The total amount of offensive play when penalties are shot after and before extra time are, respectively, $a^* + b^* = 2g^{-1}(1)$ and $a^{**} + b^{**} = g^{-1}(\gamma) + g^{-1}(1/\gamma)$. Note that $g' > 0$ implies $(g^{-1})' > 0$. Also, $1/\gamma - 1 > 1 - \gamma$ for all $\gamma \in (0, 1)$. Therefore, a sufficient condition to have $g^{-1}(\gamma) + g^{-1}(1/\gamma) > 2g^{-1}(1)$ or, equivalently, $a^{**} + b^{**} > a^* + b^*$ is $(g^{-1})'' \geqslant 0$, which can be rewritten as $g'' \leqslant 0$.

Using an analogous argument, we have:

$$h(a^*) = h(b^*) = 1 \qquad \text{and} \qquad h(a^{**}) \times h(b^{**}) = 1 \tag{9}$$

If $h(a^{**}) = \mu$ and $h(b^{**}) = 1/\mu$, then $\mu > 1$ by $h' < 0$ and Proposition 2. Again, $a^* + b^* = 2h^{-1}(1)$ and $a^{**} + b^{**} = h^{-1}(\mu) + h^{-1}(1/\mu)$. Given that $\mu - 1 > 1 - 1/\mu$ for all $\mu > 1$ and $(h^{-1})' < 0$, then a sufficient condition to have $h^{-1}(\mu) + h^{-1}(1/\mu) < 2h^{-1}(1)$ or, equivalently, $a^{**} + b^{**} < a^* + b^*$ is $(h^{-1})'' \leqslant 0$ which can be rewritten as $h'' \leqslant 0$. This conclusion is summarized as follows.

Proposition 3 (The merits of penalties before and penalties after extra time)

Assume **(C)** *holds. A sufficient condition such that penalties before extra time dominates penalties after extra time is* $g''(c) \leqslant 0$. *A sufficient condition such that penalties after extra time dominates penalties before extra time is* $h''(c) \leqslant 0$.

Two important lessons can be drawn from Proposition 3. First and on a general level, in a model with homogeneous teams and rather unrestricted functional forms of scoring probabilities, there are arguments both in favor and against having penalties shot before extra time. The debate is open. Although this inconclusive result might seem disappointing to some readers, it is in our view both surprising and interesting. Indeed, our preliminary intuition was that shooting penalties after extra time would always be optimal unless we exogenously imposed enough heterogeneity in the quality of teams. In other words, our idea was that, under team homogeneity, the distortion of strategies (one highly defensive, one highly offensive) should come at the expense of a lower average. The argument was based on the fact that increasing the level of offensive play has a positive and increasing effect on the probability of receiving a goal whereas it has a positive but decreasing effect on the probability of scoring; this, we thought, implied that the decrease in the level of offensive play by the team who won the penalty shoot-outs should be always greater than the increase in the level of offensive play by the team who lost them. Proposition 3 shows that this intuition is incorrect.

Second, although $g''(c) < 0$ and $h''(c) < 0$ involve third derivatives of the scoring probabilities, it is still possible to gain some intuitions concerning the types of situations in which penalties before extra time are likely to be beneficial or detrimental for the game. Recall that $g(c)$ denotes the ratio between the marginal effect of the strategy c in the likelihood of receiving a goal $\alpha_2(c)$ and its marginal effect in the likelihood of scoring a goal $\alpha_1(c)$. By definition, this ratio is increasing. Concavity implies that the increase is less acute as strategies become more and more offensive. As a result, if penalties are shot before extra time, the increase in the strategy by the team who lost them more than compensates for the decrease in the strategy by the team who won them. Symmetrically,

$h(c)$ denotes the inverse ratio (i.e., the marginal effect in the probability of scoring over the marginal effect in the probability of receiving a goal), which is then decreasing. Concavity of this ratio now implies that the decrease is more pronounced as the team plays more and more offensively, leading to the opposite conclusion: the decrease in the strategy of the team who won the penalty shoot-outs more than offsets the increase in the strategy of the rival.

3.2 An example

We now illustrate our result with a specific scoring function. Suppose that $\alpha(a,b)$ (and, by asymmetry, $\beta(a,b)$) belongs to the following class of functions:

$$\alpha(a,b) = \frac{1}{x}a^x + \frac{1}{y}b^y + z \qquad \text{with} \quad x \in (0,1),\ y > 1,\ z \gtreqless 0$$

By definition, we have $g(c) = \frac{c^{y-1}}{c^{x-1}} = c^{y-x}$ and $h(c) = c^{x-y}$. Note that $h'' > 0$ for all x, y and that $g'' \gtreqless 0 \Leftrightarrow y \gtreqless x+1$. Therefore, according to Proposition 3, there are no sufficient conditions such that penalties after extra time are dominant and $y < x+1$ is a sufficient condition such that penalties before extra time are dominant. Yet, we can go one step further. Given (8), $g(a^*) = 1 \Rightarrow a^* = 1$, $g(b^*) = 1 \Rightarrow b^* = 1$ and $g(a^{**}) \times g(b^{**}) = 1 \Rightarrow (a^{**})^{y-x} \times (b^{**})^{y-x} = 1 \Rightarrow a^{**} = 1/b^{**}$. As a result, and given (6), a^{**} is a root a that satisfies the following polynomial equation:

$$a^{y-x} = \frac{\frac{1}{x}a^{-x} + \frac{1}{y}a^y + z}{2 - \frac{1}{x}a^x - \frac{1}{y}a^{-y} - z} \quad \Rightarrow \quad \Big(\frac{1}{x} + \frac{1}{y}\Big)a^{y+x} + \Big(z - 2\Big)a^y + za^x + \frac{1}{x} + \frac{1}{y} = 0$$

An analytical characterization of this value is only possible under strong parameter restrictions. However, the interesting point to notice is that, independently of what the optimal values a^{**} and b^{**} are, it is *always true* that $a^{**} + b^{**} > a^* + b^*$. In fact, shooting penalties after extra time is the rule that *minimizes* the total amount of offensive play. To see why this assertion is correct, one only needs to notice that $(a^*, b^*) = (1,1)$ is the pair that solves the program $\min_{\{a,b\}} a + b$ s.t. $a = 1/b$. Naturally, the result is specific to the functional form described above. However, it highlights the general idea that a modification in the ordering of extra time and penalty shoot-outs can be an excellent candidate to increase the attractiveness of the game.

3.3 Extensions

The theory presented above could be extended in a number of directions. First, it would be interesting to generalize the more restrictive features of the model. For example, we have assumed that the rule designer maximizes the total amount of offensive play $(a+b)$. This criterion seems reasonable. However, one could think of other, such as maximization of the sum of the probabilities of scoring $(\max_{\{a,b\}}\ \alpha(a,b)+\beta(a,b))$ or minimization of the probability of deciding the winner by penalty shoot-outs $(\min_{\{a,b\}}\ (1-\alpha(a,b))(1-\beta(a,b)))$. We have also assumed team homogeneity to better isolate the incentive effects of task ordering. It would be interesting to determine whether the effects described above are mitigated or exacerbated when teams have heterogeneous qualities, and therefore heterogeneous scoring probabilities. Last, we have informally discussed how the results extend to the case of a fixed 30 minutes extra time play. However, the argument would be more solid if it were backed up by a full-fledged model.

A second, more novel alley for improvement would be to incorporate some psychological factors in the analysis. On the team side, it has been argued that players have a tendency to overestimate their chances of winning at penalty shoot-outs, with probabilities adding up to more than one. Also, teams might feel less psychological pressure if they lose at penalty shoot-outs. After all, it is just a matter of bad luck. These two considerations could provide further support for a rule change, as they tend to make penalty shoot-outs relatively less unattractive under the current system. On the spectator side, the innovative work by Chan, Courty and Hao (2003) argues that the public has a taste for games being close, which they label as a preference for suspense. This feature could very naturally be added to our model. However, it is a priori unclear in which direction the order of events would affect the demand for suspense. In a related vein, penalty shoot-outs can be interesting per se, especially for the more neutral spectator. The fact that penalties are decisive if shot after extra time but may not even count if shot before extra time makes them more exciting in the former than in the latter case. However, this advantage of the current system has to be traded-off against the possibility that the penalty shoot-out stage may never be reached.[6] This, and other psychological factors such as the aversion to sequential resolution of uncertainty by spectators rooting for a team (Palacios-Huerta, 1999) could also play a role in the comparison of systems.

The third and possibly most interesting direction for research would be to conduct a careful empirical analysis. After all, the question we ask can only be answered by looking at the data. The complication arises from the fact that the rule proposed in the paper has never been used in practice. Note however that, under a fixed 30 minutes play, shooting penalties before the extra time is similar to (although not exactly the same as) giving a one-goal advantage to one team.[7] Thus, we could compare the attack level of teams during extra time (with proxies such as shots on goal, corner kicks, etc.) when teams are tied and when one team is leading by one goal.

4 Concluding remarks

The paper has demonstrated that shooting penalties at the end of regular time and let the outcome count only if the extra time finishes with no goals has the potential to turn games more attractive. In general, it will increase the willingness to attack of the team that loses the penalty shoot-outs and increase the willingness to defend of the team that wins the penalty shoot-outs. The benefits of the first effect will dominate the costs of the second one when the marginal effect of the strategy in the probability of conceding a goal over its marginal effect in the probability of scoring increases at a lower rate the higher

[6] We thank an anonymous referee for suggesting this extension.

[7] The main difference is that a team that wins the penalty shoot-outs before extra time loses the game if it receives a goal during extra time (reward is 0), whereas a team that starts with a one-goal advantage goes to the penalty shoot-outs if it receives a goal during extra time (reward is 1/2).

the initial level of offensive play.

We would like to conclude with a more general note. In our view, sports is an excellent candidate for empirical and theoretical analyses of games and strategies. Empirical tests of game theory, incentive theory and contract theory using data from sports were uncommon in the past, despite the availability, suitability and quality of data. In recent years, however, researchers have acknowledged and employed this source of information. Studies have been conducted to test basic predictions of game theory such as mixed strategy play (Chiappori, Levitt and Groseclose (2002) and Palacios-Huerta (2003) using soccer data and Walker and Wooders (2001) using tennis data) and also predictions of incentive theory such as effort in a career-concerns environment (Wilczynski (2003) using NBA data). As mentioned in the introduction, there have also been some recent attempts to test basic effects of rule changes on the behavior of teams (Palacios-Huerta (2004), Aiello and Veall (2003) and Abrevaya (2004)). On the theory side, efforts to understand some slightly more sophisticated effects of rules on strategies have not followed the same pace (Palomino, Rigotti and Rustichini (2000) and Brocas and Carrillo (2004) are some exceptions). In our opinion, this is unfortunate. Given the clarity, simplicity and malleability of rules, interesting (yet non-trivial) insights can be obtained with a careful modelling and a rigorous application of game theoretic tools. These models could be used to provide clear-cut (yet non-obvious) welfare and policy implications. We view this paper as a modest attempt in that direction. We hope that it will stimulate further research on the topic and that Sports Organizations such as FIFA, UEFA, NBA, NHL, etc. will realize the value of such studies.

References

1. Abrevaya, J. (2004) "Fit to be Tied: The Incentive Effects of Overtime Rules in Professional Hockey", *Journal of Sports Economics*, 5, 292-306.

2. Aiello, M., and M. Veall (2003) "The Effects of Changing the Incentives to Tie: Evidence from Two Sports", *mimeo*, McMaster University.

3. Brocas, I., and J.D. Carrillo (2004) "Do the 'Three-Point Victory' and 'Golden Goal' Rules Make Soccer More Exciting?", *Journal of Sports Economics*, 5, 169-185.

4. Chan, W., Courty, P. and L. Hao (2003) "Suspense", *mimeo*, London Business School.

5. Chiappori, P.A., Levitt, S. and T. Groseclose (2002) "Testing Mixed Strategy Equilibria when Players are Heterogeneous: the Case of Penalty Kicks in Soccer", *American Economic Review*, 92(4), 1138-1151

6. Lazear, E.P. and S. Rosen (1981) "Rank-Order Tournaments as Optimum Labor Contracts", *Journal of Political Economy*, 89(5), 841-864.

7. Palacios-Huerta, I. (1999) "The Aversion to the Sequential Resolution of Uncertainty", *Journal of Risk and Uncertainty*, 18(3), 249-269.

8. Palacios-Huerta, I. (2003) "Professionals Play Minimax", *Review of Economic Studies*, 70(2), 395-415.

9. Palacios-Huerta, I. (2004) "Structural Breaks During a Century of the World's Most Popular Sport", *Statistical Methods and Applications*, 13(2), 241-258.

10. Palomino, F., Rigotti, L. and A. Rustichini (2000) "Skill, Strategy and Passion: an Empirical Analysis of Soccer", *mimeo*, Tilburg University.

11. Prendergast, C. (1999) "The Provision of Incentives in Firms", *Journal of Economic Literature*, 37(1), 7-63

12. Walker, M. and J. Wooders (2001) "Minimax Play at Wimbledon", *American Economic Review*, 91(5), 1521-38.

13. Wilczynski, A. (2003) "Career Concerns and Renegotiation Cycle Effect", *mimeo*, U. of Chicago.